Financial Engineering: A Flexible Longevity Bond to Manage Individual Longevity Risk

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Background and Motivation

Individuals have increasing responsibility to manage their own longevity risk

- Global trend of moving from defined benefit (DB) to defined contribution (DC) pension plans (Willis Towers Watson, 2018)

Thin life annuity markets in Australia and worldwide and so-called annuity puzzle (Modigliani, 1986)

- Demand Side: high loadings, bequest motives, liquidity and loss aversion (Brown, 2009); Supply Side: low interest rates, interest rate risk, longevity risk, limited ability to hedge longevity risk (Evans and Sherris, 2010)

Innovation required in longevity risk product design to:

- Change focus from insurance product (life annuity) to investment product (bond) for individual longevity risk

- Allow flexible selection of bond income level and principal repayment as a death benefit (bequest motive)

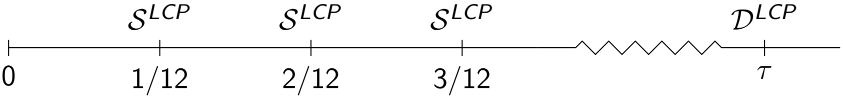
What we do

- Present the financial engineering valuation and immunizing framework for a new individual longevity bond, fully collateralized with government bonds
- Calibrate and apply recent state-of-the-art continuous-time AFNS interest rate and mortality models for systematic mortality risk (Blackburn and Sherris, 2013; Xu et al., 2019; Huang et al., 2019)
- Apply immunization theory with linear programming and a mean-absolute deviation constraint (Liu and Sherris, 2017)
- Compare and assess immunized bond portfolios (coupon bonds and annuity bonds) for the individual longevity bonds with Australian government bonds
- Price aggregate mortality risk using Australian population mortality to determine bond price loadings, and quantify natural hedging in bond design.

Longevity Bond Designs - Basis Bonds

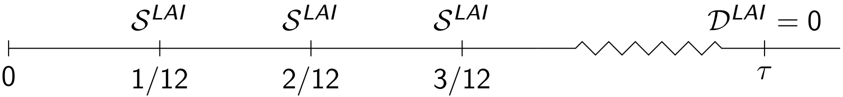
Lifetime coupon and principal bond (LCP Bond)

- Monthly coupon payments while alive: $\mathcal{S}^{LCP} = \frac{r_c}{12} \times \mathcal{D}^{LCP}$
- Full principal return on death: $\mathcal{D}^{LCP}$



Lifetime annuity income bond (LAI Bond)

- Monthly coupon payments while alive: $\mathcal{S}^{LAI}$
- No principal return on death: $\mathcal{D}^{LAI} = 0$

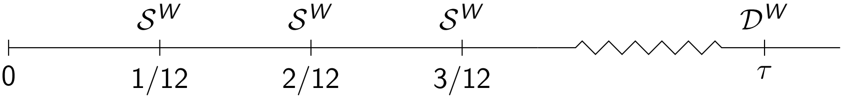


Longevity Bond Designs - Flexible Bond

Flexible lifetime income and capital bond series W% (Flexible LIP Bond^W)

- $W\%$ of Lifetime coupon and principal bond (LCP Bond)
- $(100 - W)\%$ of Lifetime annuity income bond (LAI Bond)

$$\begin{aligned}
\text{Survival Benefit: } \mathcal{S}^W &= W\% \cdot \mathcal{S}^{LCP} + (1 - W\%) \cdot \mathcal{S}^{LAI}, \\
\text{Death Benefit: } \mathcal{D}^W &= W\% \cdot \mathcal{D}^{LCP} + (1 - W\%) \cdot \mathcal{D}^{LAI} \\
&= W\% \cdot \mathcal{D}^1.
\end{aligned}$$



Interest Rate Risk - Arbitrage-Free Nelson-Siegel Model

Arbitrage-Free Nelson-Siegel model (AFNS) with Level (L) , Slope (S) and Curvature (C) Factors (Christensen et al., 2011). Widely used model with both theoretical and empirical strengths.

Yield to maturity (affine in factors)

$$
\begin{aligned}
y(t,T) =& L_t + S_t\left(\frac{1-e^{-\delta(T-t)}}{\delta(T-t)}\right) + C_t\left(\frac{1-e^{-\delta(T-t)}}{\delta(T-t)} - e^{-\delta(T-t)}\right) - \frac{A(t,T)}{T-t}, \\
=& -\frac{B(t,T)^\top}{T-t}X_t - \frac{A(t,T)}{T-t}.
\end{aligned}
$$

where $X_t = (L_t, S_t, C_t)^\top$. The present value factor for pricing (affine in risk factors):

$$
D(t,T) = e^{-(T-t)y(t,T)} = e^{B(t,T)^\top X_t + A(t,T)},
$$

Calibrated to Australian daily zero-coupon interest rates from 1992 to 2018, source - Reserve Bank of Australia, using Kalman filter and MLE.

Interest Rate Risk - Arbitrage-Free Nelson-Siegel Model

Yield curve is consistent with and derived from the dynamics of the risk factors. Yield curve parameters determined from the fitted factor parameters and satisfy an arbitrage-free requirement. Dynamics for the factors - follow stochastic differential equation (SDE):

$$\mathrm{d}X_t = K^{\mathrm{Q}}\left[\theta^{\mathrm{Q}}(t) - X_t\right]\mathrm{d}t + \Sigma \mathrm{d}W_t^{\mathrm{Q}},$$
$$\mathrm{d}X_t = K^{\mathrm{P}}\left[\theta^{\mathrm{P}}(t) - X_t\right]\mathrm{d}t + \Sigma \mathrm{d}W_t^{\mathrm{P}},$$

where $X_t = (L_t, S_t, C_t)^\top$,

$$K^{\mathrm{Q}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \delta & -\delta \\ 0 & 0 & \delta \end{pmatrix}, K^{\mathrm{P}} \begin{pmatrix} k_{11}^{\mathrm{P}} & 0 & 0 \\ 0 & k_{22}^{\mathrm{P}} & 0 \\ 0 & 0 & k_{33}^{\mathrm{P}} \end{pmatrix}, \Sigma = \begin{pmatrix} \sigma_{11} & 0 & 0 \\ 0 & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{pmatrix}.$$

Continuous time equivalent of auto-regressive time series model. P and Q measures differ by price of risk $dW_t^Q = dW_t^P + \Lambda_t dt, \Lambda_t = \lambda^0 + \lambda^1 X_t$.

Interest Rate Risk - AFNS Interest Rate Model Goodness of Fit

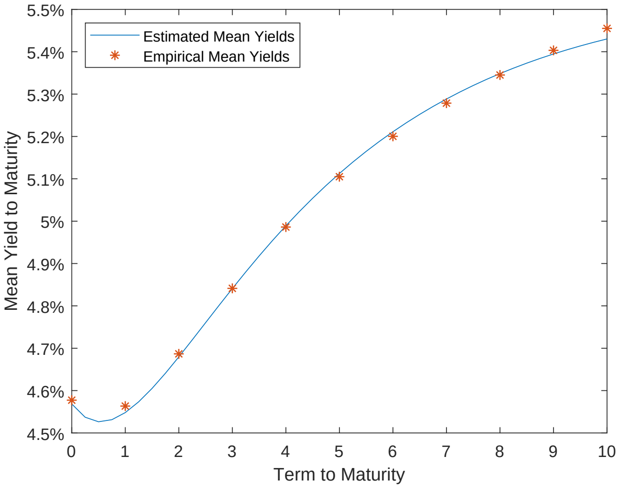


Figure 1: Empirical and Estimated Mean Yield Curves.

Interest Rate Risk - Yield Curve Simulation Results

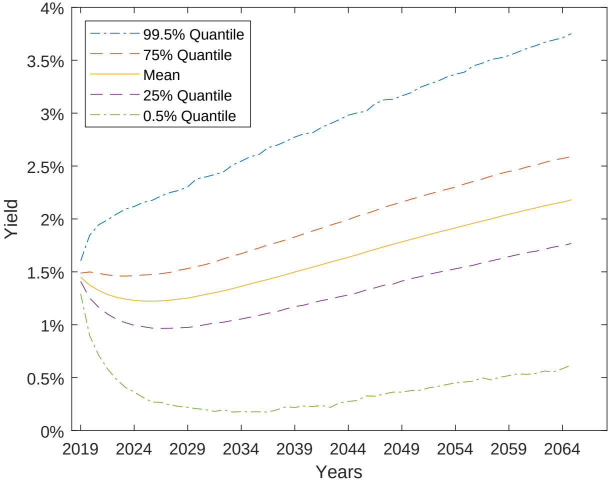


Figure 2: Mean One-Month Yield-to-Maturity with the 25%, 75%, 0.5%, and 99.5% Quantiles of 20000 Simulations.

Longevity Risk: AFNS Mortality Model

Mortality model is a consistent affine continuous time AFNS mortality with factors for Level (L), Slope (S) and Curvature (C). (Blackburn and Sherris, 2013; Xu et al., 2019; Huang et al., 2019)

$$y(t, T) \longrightarrow \overline{\mu}(t, T)$$
$$\theta^P(t) \longrightarrow 0$$

The survival probability of individual age x from t to T is:

$$S\left(t, T\right) = e^{-(T-t)\overline{\mu}(t,T)} = e^{B(t,T)^\top X_t + A(t,T)}$$

Data: Australian male mortality for cohorts born from 1856 to 1907, obtained from Human Mortality Database. Fitted with Kalman filter and MLE.

Model is an age-cohort for Australian male mortality using 1856 to 1907 cohorts with full observations of cohort mortality rates from age 65 to 110. Calibrated to historical data.

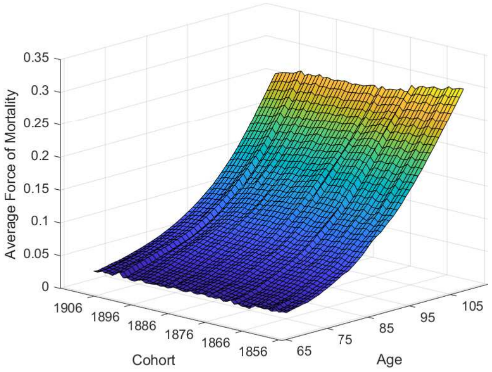


Figure 3: Australian Cohort Average Force of Mortality for Males Born between 1856 and 1907, from Age 65 to 110.

Longevity Risk: AFNS Mortality Model

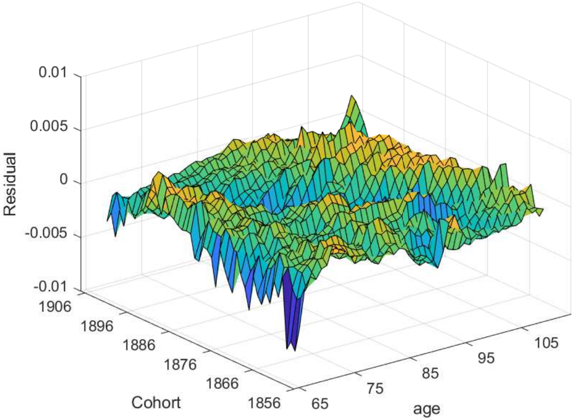
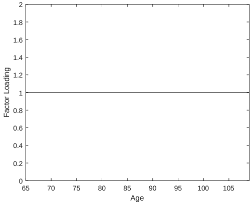
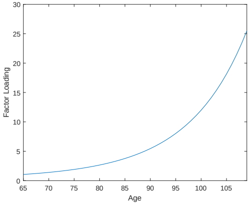


Figure 4: Residuals of the AFNS Mortality Model.

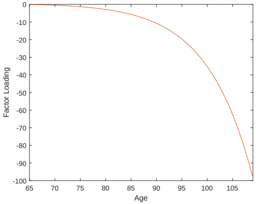
Longevity Risk: AFNS Mortality Model



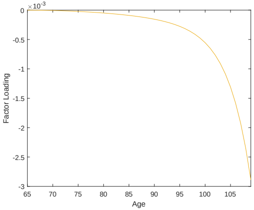
(a) $B_1 = -\frac{B^1(t,T)}{T-t}$.



(b) $B_2 = -\frac{B^2(t,T)}{T-t}$.



(c) $B_3 = -\frac{B^3(t,T)}{T-t}$.



(d) $A = -\frac{A(t,T)}{T-t}$.

Figure 5: Factor Loadings of the AFNS Mortality Model.

AFNS Mortality Model Goodness of Fit

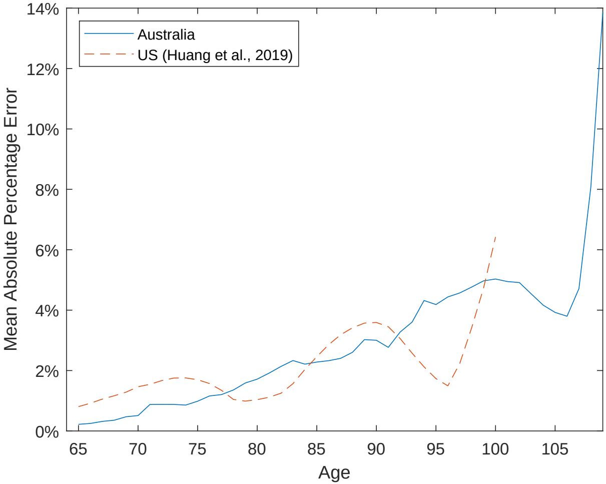


Figure 6: MAPE of the Survival Curve of the AFNS Mortality Model.

AFNS Mortality Model - Force of Mortality Simulation

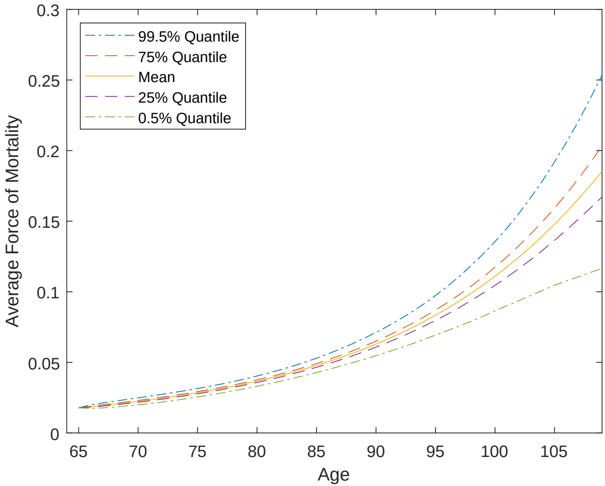


Figure 7: Mean of the Average Forces of Mortality with the 25%, 75%, 0.5%, and 99.5% Quantiles of 20000 Simulations.

Bond Pricing and Immunization

- We assess individual longevity bonds issued to an Australian male aged 65 on 01/Jan/2019 with year of birth 1954, and maximum attainable age 110. So that the modeling period is from 2019 to 2064.
- For illustration bonds are assumed to have an annual coupon rate of $r_c = 2\%$ paid monthly and all bonds are priced to be 100 at issue.
- Bonds are priced as discounted expected present value of the monthly cash flows paid on survival and death using the AFNS interest rate and mortality models.
- Mortality assumptions for the individual bonds are based on aggregate population mortality - in practice this is adjusted for adverse selection - expect higher mortality than aggregate for LCP Bond and lower mortality than aggregate for LAI Bond.
- With bequest motives, expect most individuals to select a flexible mix so that adverse selection is limited and mitigated by natural hedging in the individual longevity bond cash flows.

Individual Longevity Bond Expected Cash Flows

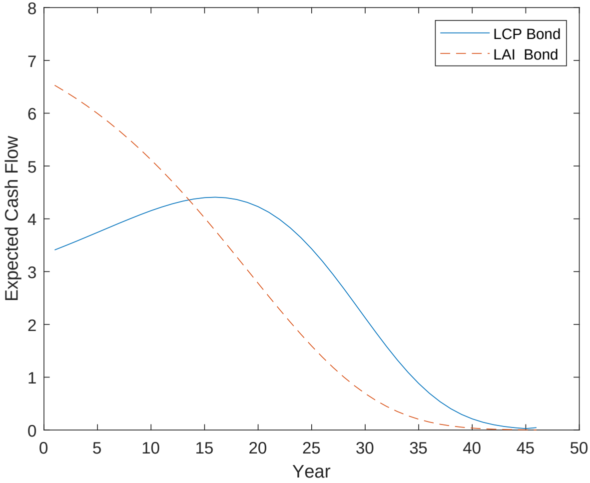


Figure 8: Expected Cash Flows 20000 Simulations, $r_c = 2\%$.

Immunization - Government Bond Portfolio

Immunized portfolio determined using linear programming:

$$\text{Net Portfolio Cash Flow (NP)} = \text{Asset Cash Flow} - \text{Liability Cash Flow}$$

$$\max_{\omega}(Conv_{NP})$$

subject to:

$$Dur_{NP} = 0,$$
$$Value_{NP} = \sum_t n_t = 0,$$
$$\sum_{t>0} n_t \times (t-h)^+ \leq 0, \text{ for all positive h},$$

where

$$n_t = \sum_i \omega_i \times EPV(\text{CF}_{i,t}) - EPV(\text{CF}_{LB,t}).$$

Government Bonds in Immunizing Portfolio - Coupon Bonds and Annuity Bonds

Coupon Bonds:

- Australian government coupon bonds - TTM 1 to 29 years

Annuity Bonds:

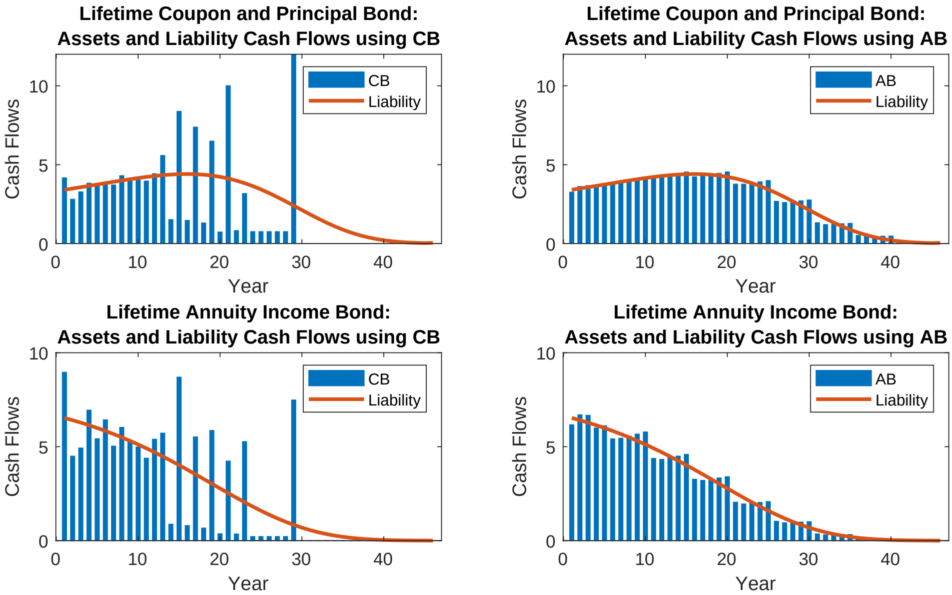
- Based on NSW government Waratah annuity bonds, Monthly inflation-indexed payments, with existing:TTM 3 to 5 years
- Hypothetical Bonds: Issued in Jan 2019 with TTM of 5, 10, 15, 20, 25, 30, 40 years and monthly fixed annuity payments.

Individual Longevity Bonds - Duration, Convexity and VaR

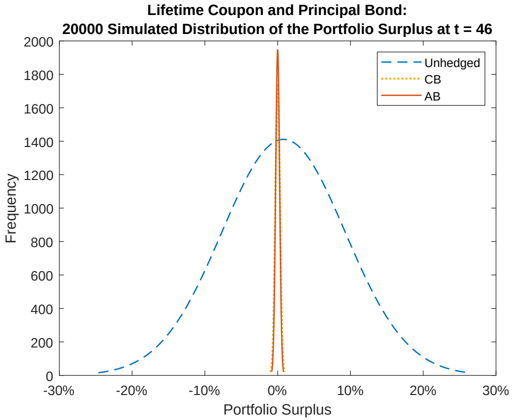
Table 1: Duration, Convexity and VaR.

	Price	Fisher Dur	Fisher Conv	$VaR_{0.5\%}$
Lifetime coupon and principal bond	100	14.70	298.44	-26.47%
Lifetime annuity income bond	100	10.37	165.82	-14.71%

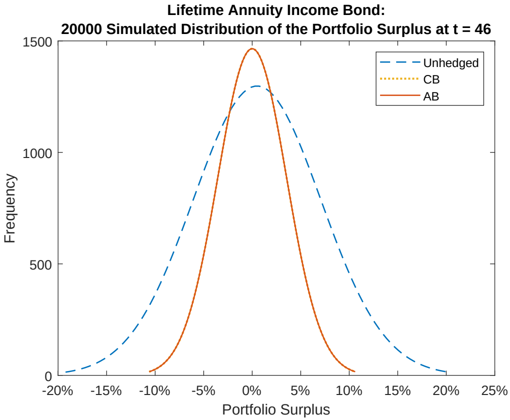
Immunized Bond Portfolios - Coupon Bonds compared with Annuity Bonds



Immunized Bond Portfolios - Coupon Bonds compared with Annuity Bonds



(a) LCPB



(b) LAIB

Figure 10: Final Year Surplus Distribution of Individual Bonds, Portfolio Immunized with only Coupon Bonds and only Annuity Bonds.

Immunized Bond Portfolios - Coupon Bonds compared with Annuity Bonds

Table 2: Comparison of Final Year Surplus Distribution $\mathrm{VaR}_{0.5\%}$.

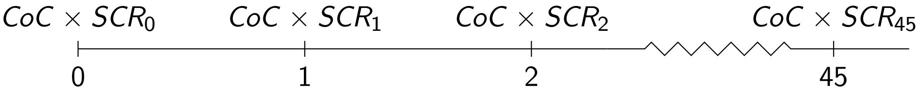
	Individual Bonds	Immunized with CB	Immunized with AB
$\mathrm{VaR}_{0.5\%}$(LCP Bond)	-26.47%	-0.95%	-0.89%
$\mathrm{VaR}_{0.5\%}$(LAI Bond)	-14.71%	-9.92%	-9.91%

- For the immunized portfolios, the VaR can be used to determine the price loading for systematic longevity risk and the residual interest rate risk.
- Based on final year surplus, for the LCP Bond a loading of approximately 1% and for the LAI Bond a loading of approximately 10% and the Flexible Bonds a weighted average of these.
- Benefits of natural hedging in the LCP Bond are significant.

Pricing Systematic Longevity Risk - Capital Requirements for SPV

The capital requirements for systematic longevity risk under Solvency II, based on the standard formula for SCR:

$$SCR_t := NAV_t - (NAV_t | \text{Mortality Shock } 0.5\%),$$



We quantify the cost of longevity risk with risk margin (RM):

$$RM = \sum_{t=0} CoC \times SCR_t \times D(0,t), \qquad \text{where } CoC = 6\% \text{ under Solvency II.}$$

Capital requirement as a proportion of the premium to cover the RM:

$$Loading = \frac{RM}{Price}.$$

Pricing Systematic Longevity Risk - Risk Margin and Loading

Table 3: Loading for LCP Bond and LAI Bond based on CoC and Regulatory Capital

	Loading_{CB}	Loading_{AB}
Lifetime Coupon and Principal Bond (W=100)	5.08%	5.02%
Lifetime Annuity Income Bond (W=0)	6.88%	6.87%

- Flexible individual longevity bond loadings are a weighted average of these two bond loadings.
- Benefits of natural hedging lower for CoC and Regulatory Capital compared to final year surplus.
- If loadings in practice reflect Regulatory Capital then LCP Bond more profitable based on final year surplus distribution.

Summing Up

- Proposed and assessed a longevity bond for individuals as a post-retirement investment product not currently available in the market:
  - Flexible structure allowing for bequest and liquidity preferences with built-in natural hedging
  - Priced using state-of-the-art financial engineering for interest rate and mortality risk
  - Immunized with a fully collateralized government bond portfolio
- Presented and calibrated AFNS interest rate and mortality models with Australian data, used for pricing and simulation.
- Quantified the effectiveness of the immunizing bond portfolio using net cash flow and portfolio surplus - coupon bonds as effective as annuity bonds.
- Quantified and priced the longevity risk using final year surplus and Regulatory Capital to assess required bond price loadings for the individual bonds.

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Thank you for your attention

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