Predicting Cyber-Attacks using Hawkes processes

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RESEARCH INITIATIVE
Cyber Risk: Actuarial Modeling

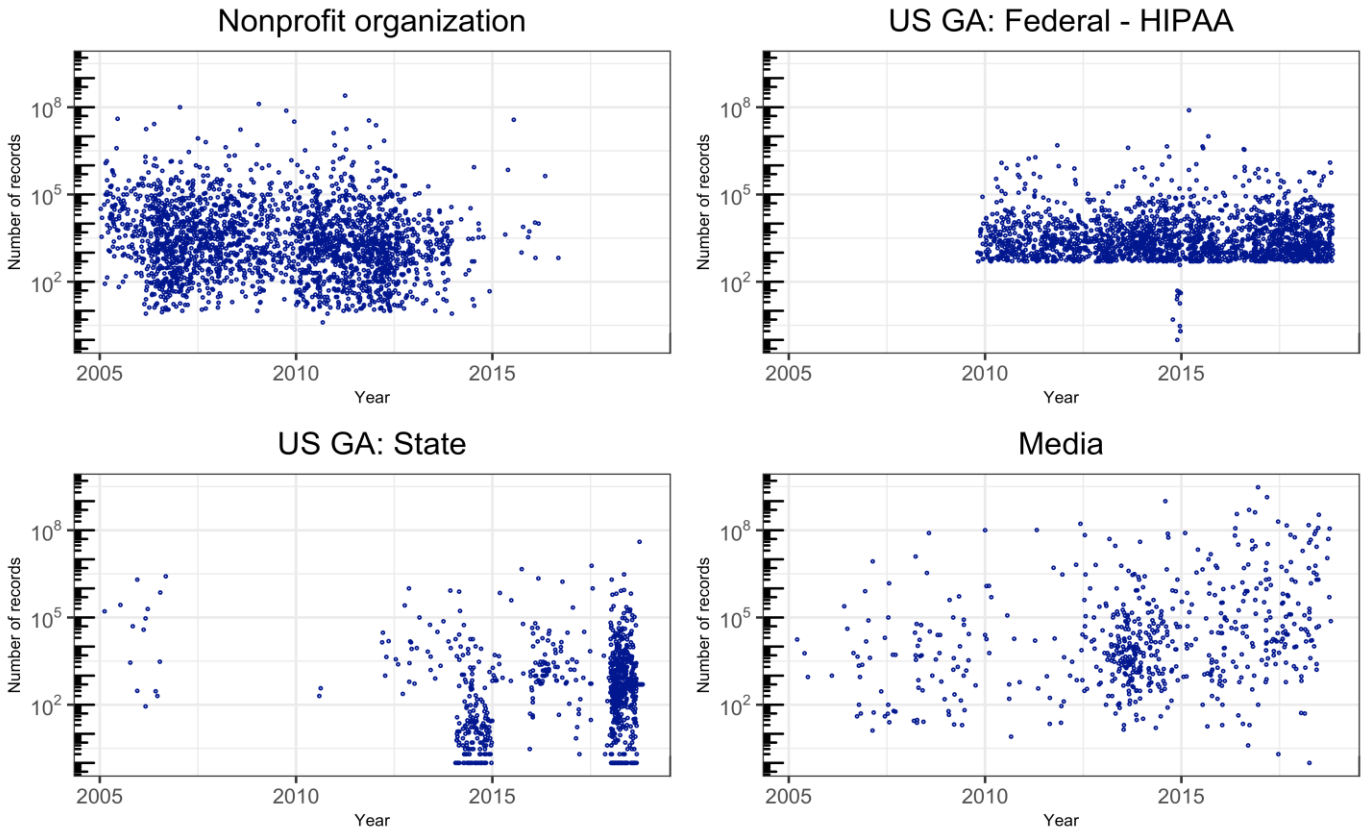
Outline of the Talk

- Data breaches dataset
- Hawkes model
- Fitting and prediction

Data breaches dataset

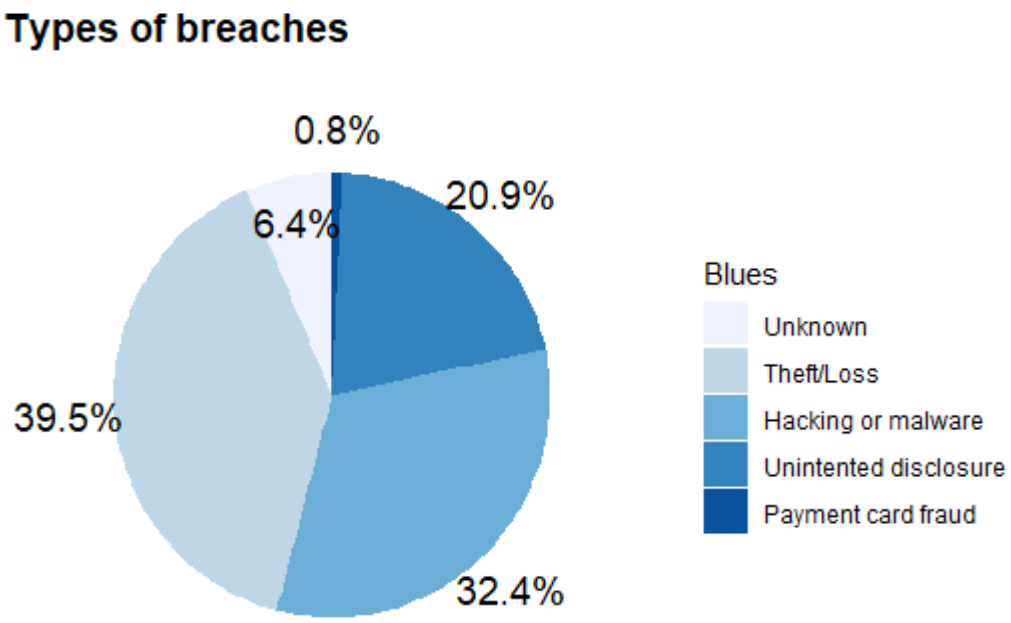
Privacy Rights Clearinghouse

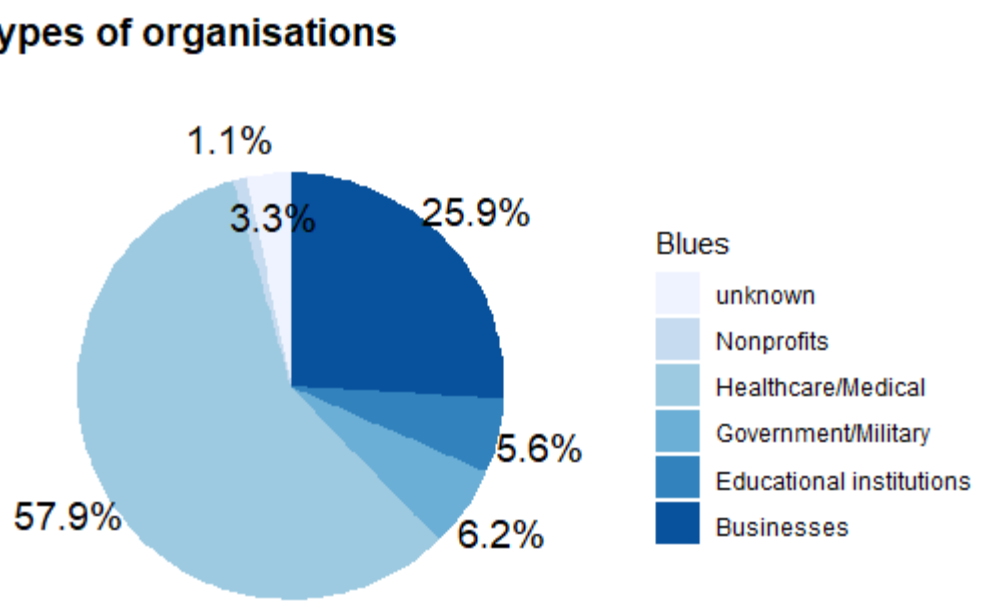
- A public database that contains 8800 data breaches in the US over the period 2005-2019
- Different types of sources reporting the cyber breaches to the database



Data breaches dataset

Descriptive statistics over 2010-2018



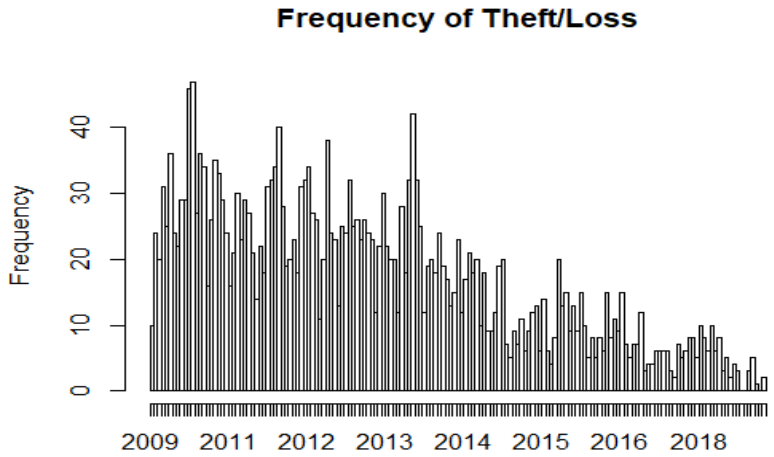


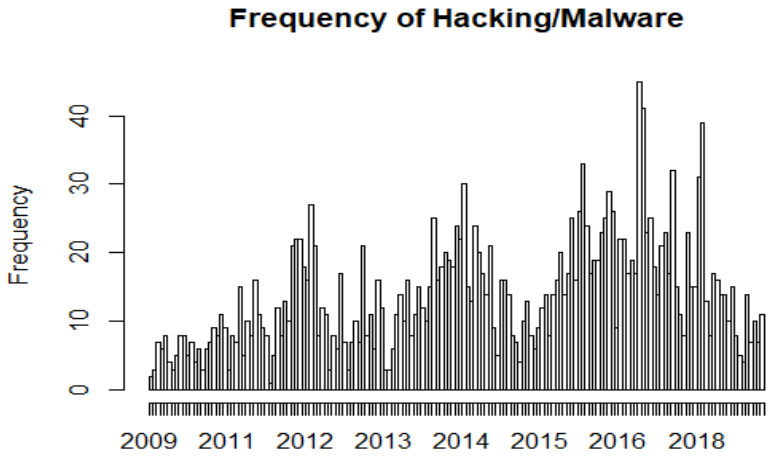
- A majority of **Theft/Loss** and **Hacking/Malware**
- 21% of Unintented disclosure

- A majority in **Healthcare/Medical**
- Businesses are well represented too

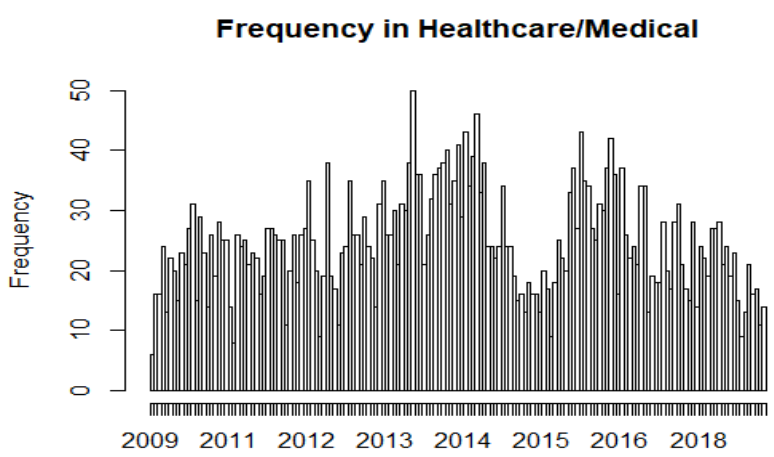
Data breaches dataset

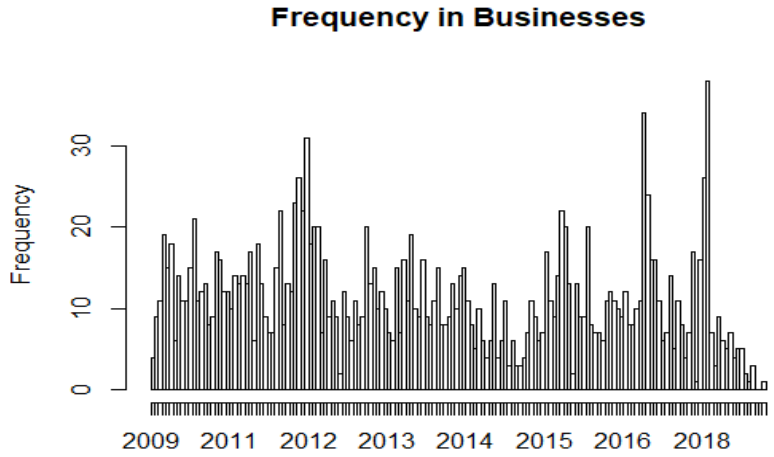
Cyber attacks frequencies by type and organization





- Apparent **clustering** by type of attacks
- Deterministic **trends** or stochastic regimes?



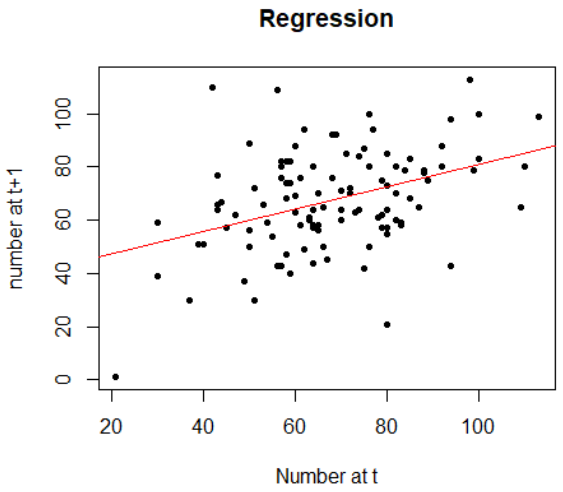


- Apparent **clustering** by type of organization attacked
- No clear **trends**

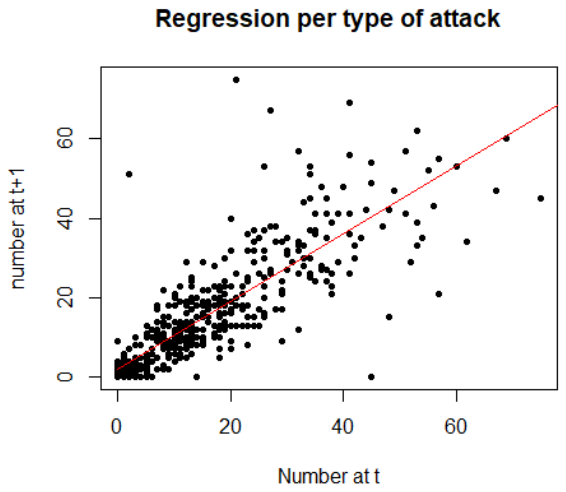
Data breaches dataset

Autocorrelation of the number of incidents

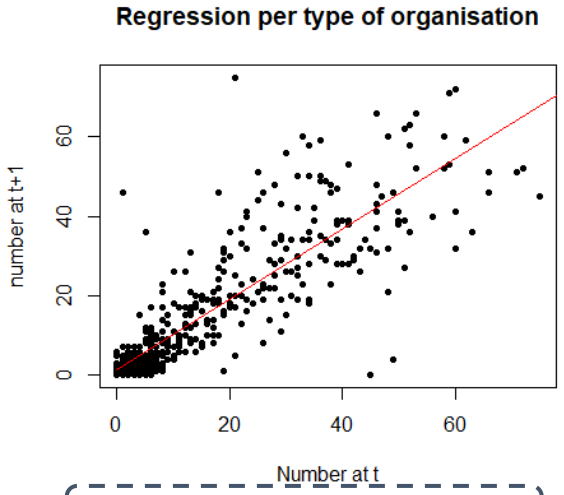
- Regression of the number of event during the **following month $t+1$** as a function of the number of event during the current month t → should be independent for a Poisson process model to be valid
- Autocorrelation dramatically increases when focusing on attacks and/or organizations of **the same type**



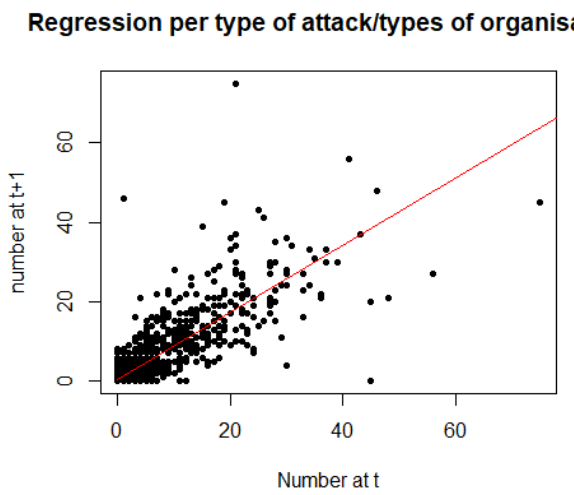
- R-squared : 0.154
- Confidence interval (95%) $[0.030, 0.278]$



- R-squared : 0.726
- Confidence interval (95%) $[0.687, 0.766]$



- R-squared : 0.780
- Confidence interval (95%) $[0.750, 0.810]$



- R-squared : 0.718
- Confidence interval (95%) $[0.702, 0.735]$

Hawkes model

Choice of the Hawkes model

- **Taking into account autocorrelation**
  - Cox model : Poisson model with stochastic intensity → difficulty to specify the stochastic intensity dynamics
  - Hawkes model : Self-exciting model with stochastic intensity, fully specified by the point process itself

- **Choice of the Hawkes model:**
  - Self-excitation: every event increases the probability for a new event to occur within a given group (same organization or attack type) → Clustering
  - Inter-excitation: in the case of multi-dimensional Hawkes process, every attack in one group increases the occurrence probability of new events in the other groups

- **Related references:**
  - Giesecke et al. (2010), Bacry et al. (2015) (finance)
  - Peng et al. (2017), Baldwin et al. (2017) (cyber risk)

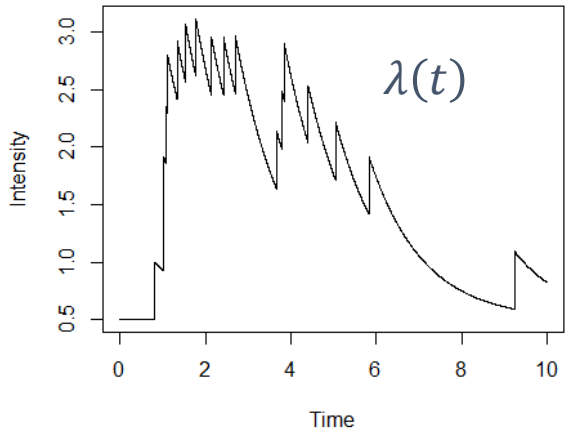
Univariate Hawkes process

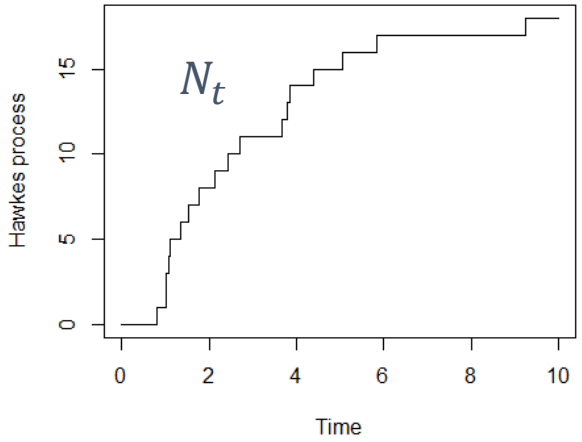
- An univariate Hawkes process with **exponential kernel** is a counting process $N_t = \sum_{n\geq 1} 1_{T_n \leq t}$ with intensity:

$$\lambda(t) = \mu(t) + \sum_{T_n<t} \alpha \exp\left(-\beta(t - T_n)\right)$$

$\mu: \mathbb{R}_+ \to \mathbb{R}_+$ is a deterministic baseline intensity

- The sum represents the **impact of past events;** it captures the **self-excitation property**





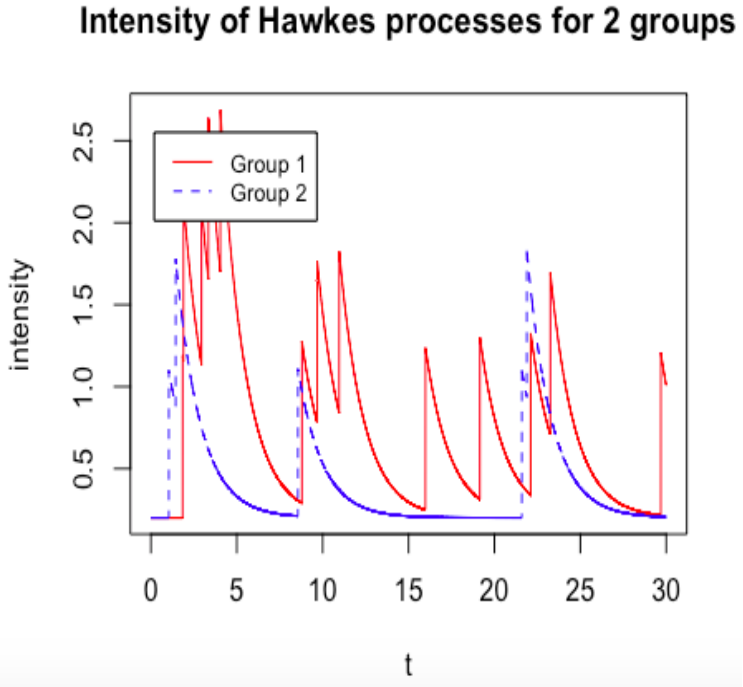
- **Each jump** represents an attack
- **Clustering phenomena**
- Intensity decreases **exponentially** between jumps

Multivariate Hawkes process

- Multivariate Hawkes process allows to model interactions **between types of entities/attacks/states**:
  $\left(N_t^{(1)}\right)_{t\geq 0}, \ldots, \left(N_t^{(K)}\right)_{t\geq 0}$, K counting processes with **jump times** $\left(T_n^{(1)}\right)_{n\geq 1}, \ldots, \left(T_n^{(K)}\right)_{n\geq 1}$
  The **intensity process** with exponential kernel of the counting process (i) is defined as:

$$\lambda_i(t) = \mu_i(t) + \sum_{j=1}^{K} \sum_{T_n^{(j)} < t} \alpha_{i,j} \exp\left\{-\beta_{i,j}(t - T_n^{(j)})\right\}$$

$\alpha_{i,j}$, $\beta_{i,j}$: **Impact of group j on group i**



Group 1 self-excitation

Impact of Group 2 on Group 1

Matrix of excitation:

$$\alpha = \begin{bmatrix} \alpha_{1,1} & \alpha_{1,2} \\ \alpha_{2,1} & \alpha_{2,2} \end{bmatrix} = \begin{bmatrix} 0.0 & 0.99 \\ 0.0 & 0.90 \end{bmatrix}$$

- Group 2 is **purely self-excited**
- Group 1 is **fully influenced by Group 2**

Kernels of multivariate Hawkes process

- « Classical » exponential kernel:

$$\phi_{i,j}(s) = \alpha_{i,j} \exp(-\beta_{i,j} s)$$

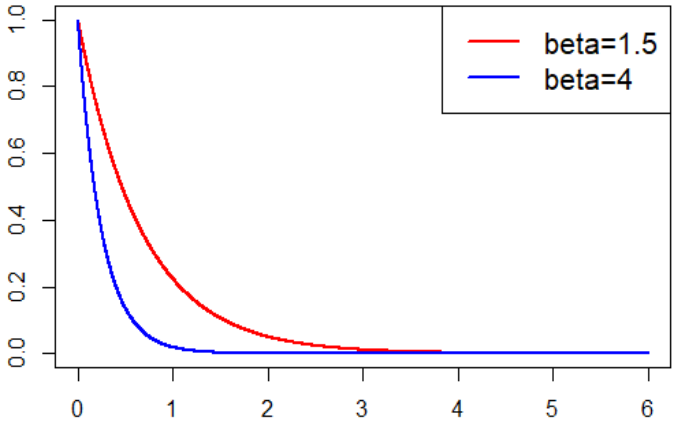
  - Instantaneous excitation
  - **Complexity**: we assume all $\beta_{i,j}$ (excitation memory of i from j) depend on the groups and are to be calibrated
  - The intensity process is not Markov (for dimension ≥ 2)

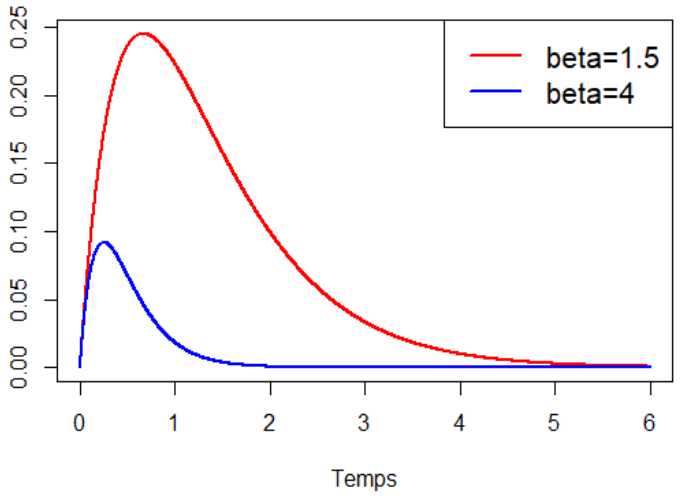
- Kernel with **delay**:

  - $\phi_{i,j}(s) = \alpha_{i,j} s \exp(-\beta_i s)$
  - The intensity process is not Markov (even in dimension 1)
  - **Complexity**: we assume all β_i (excitation memory of i from any other group) depend on the groups and are to be calibrated

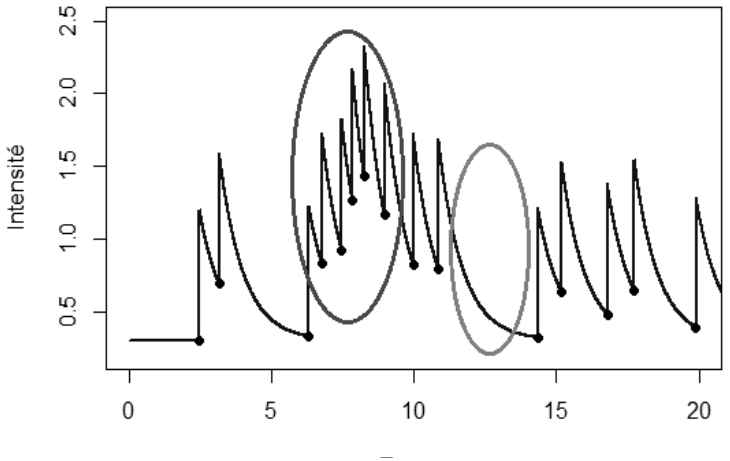
Development of closed-form formulas for the expected number of claims for such multivariate Hawkes processes

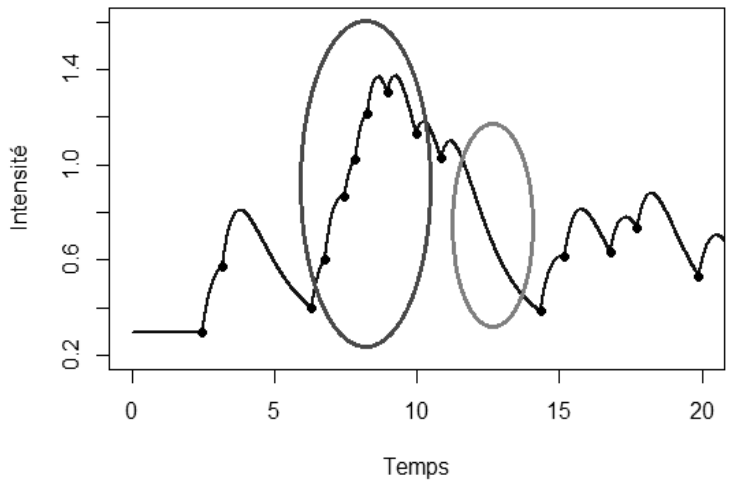
Kernels





Intensity process





Fitting and prediction

Data grouping

- **Crossing variables: attack type, sector, state**
  - Retaining groups with more than 200 attacks and remaining in OTHER
  - Total: **six groups**

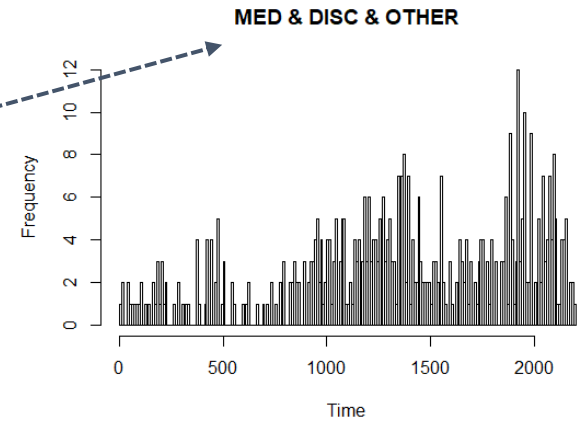
Group	Number of breaches
OTHER (1)	2046
MED & DISC & OTHER (2)	497
BUSINESSES & HACK & OTHER (3)	386
MED & HACK & OTHER (4)	472
MED & THEFT/LOSS & CALIFORNIA (5)	214
MED & THEFT/LOSS & OTHER (6)	943

Model specification

- **The three Hawkes kernels considered**

	Kernel 1 (exp)	Kernel 2 (exp)	Kernel 3 (delay)
$\phi_{i,j}(s)$	$\alpha_{i,j}\exp(-\boldsymbol{\beta_i} s)$	$\alpha_{i,j}\exp(-\boldsymbol{\beta_{i,j}} s)$	$\alpha_{i,j}\boldsymbol{s}\exp(-\boldsymbol{\beta_i} s)$
Nb parameters	54	84	54

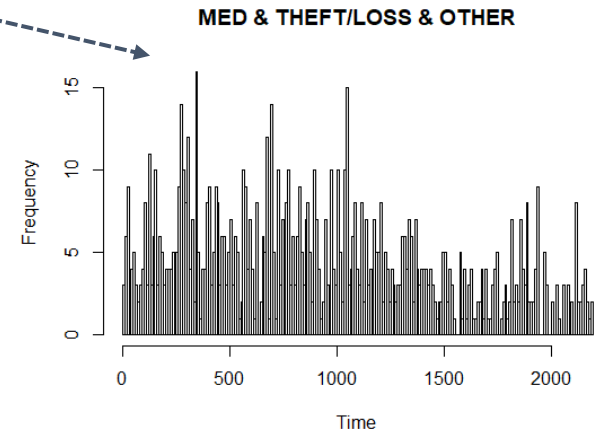
- **Baseline intensity**: $\boldsymbol{\mu_i(t) = \mu_{0,i} + \gamma_i t}$ to account for **trends** in the dataset



- **Possibility to add a Lasso penalty** (not discussed here)

$$L(\mu,\phi)_{penalized} = L(\mu,\phi) - \boldsymbol{\nu} \sum_{\boldsymbol{1\le i,j\le d}} |\boldsymbol{\alpha_{i,j}}|$$

  - Reduce complexity
  - Improve prediction capacity



Calibration results

- **Likelihood:** kernel 3 with delay best fits the data

	Kernel 1 (exp)	Kernel 2 (exp)	Kernel 3 (delay)
$\phi_{i,j}(s)$	$\alpha_{i,j}\exp(-\boldsymbol{\beta_i} s)$	$\alpha_{i,j}\exp(-\boldsymbol{\beta_{i,j}} s)$	$\alpha_{i,j}\boldsymbol{s}\exp(-\boldsymbol{\beta_i} s)$
Nb parameters	54	84	54
-Likelihood (2011-2015)	6513	6172	**6153**
-Likelihood (2011-2016)	7639	7516	**7485**

- **Adequacy tests (Kolmogorov-Smirnov):** adequacy is satisfactory, except for group (4)

OTHER (1)	**0.0503**	**0.0865**	**0.9060**
MED & DISC & OTHER (2)	**0.5546**	**0.1300**	**0.5173**
BUSINESSES & HACK & OTHER (3)	**0.5558**	**0.5966**	**0.3363**
MED & HACK & OTHER (4)	0.0024	0.0361	0.0370
MED & THEFT/LOSS & California (5)	**0.1146**	**0.5669**	**0.4246**
MED & THEFT/LOSS & OTHER (6)	**0.0733**	**0.6341**	**0.5379**

Calibration analysis

Captures the baseline non-excited intensity

Same orders of magnitude

captures the historical trend

	$\mu_0^{(i)}$	β_i	γ_i
OTHER (1)	0.87	5.39	-2.53e-04
MED & DISC & OTHER (2)	0.02	6.88	9.52e-05
Businesses & HACK & OTHER (3)	0.12	7.31	-3.56e-06
MED & HACK & OTHER (4)	0.02	5.75	9.65e-05
MED & Theft/Loss & CALIFORNIA (5)	0.05	5.96	-7.26e-06
MED & Theft/Loss & OTHER (6)	0.36	5.84	-1.07e-04

Table 7: Parameters $(\mu_0^{(i)})_{1\leq i\leq 6}$, $(\beta_i)_{1\leq i\leq 6}$ and $(\gamma_i)_{1\leq i\leq 6}$

	1	2	3	4	5	6
1	6.04	6.06	4.36	3.51	2.54	2.95
2	1.48	6.28	1.82	4.70	3.31	0.83
3	1.45	1.34	3.17	1.84	0.14	1.15
4	0.31	2.83	1.74	8.37	0.32	0.12
5	0.38	0.62	0.12	1.19	7.80	0.99
6	2.03	2.57	3.15	1.63	0.83	6.70

Captures the major self/external interactions

Table 8: Parameters $(\alpha_{i,j})_{1\leq i,j\leq 6}$

Strong reciprocal self-excitation of groups (2) and (4)

«Causal» excitation of (2) by (5)

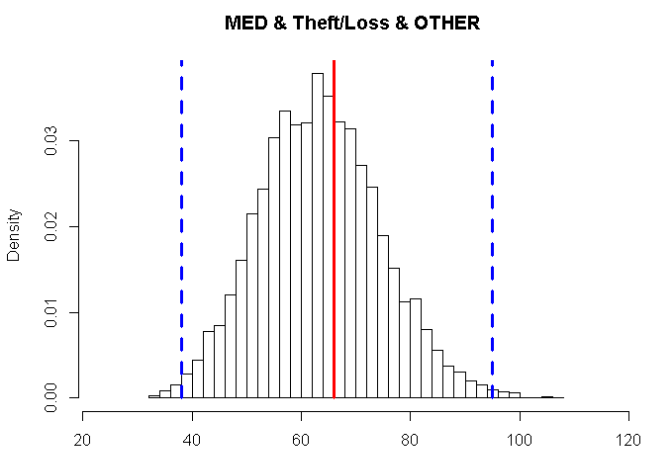
Strong self-excitation in these MED groups

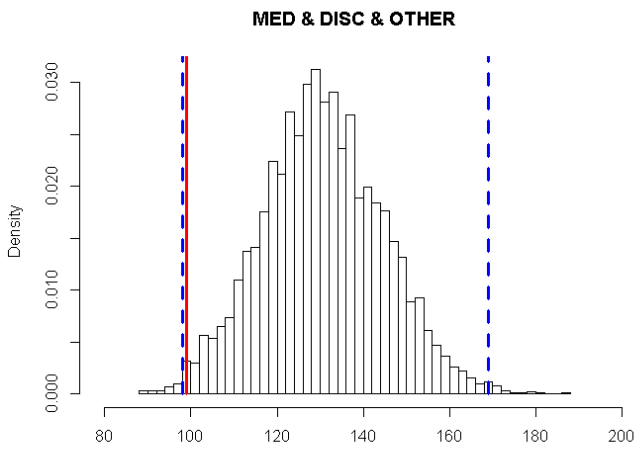
Matrix of ratios between maximal excitation and baseline intensity:

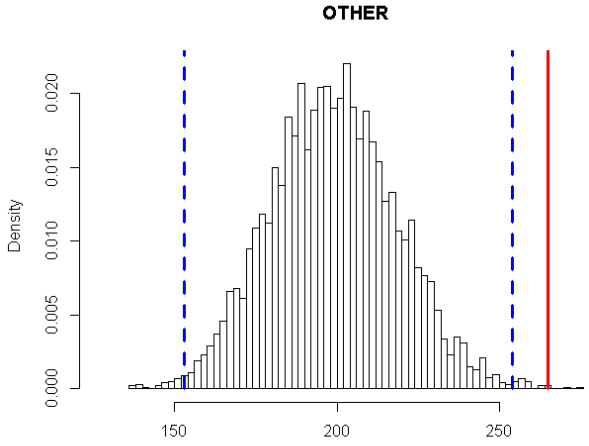
	1	2	3	4	5	6
1	0.47	0.47	0.3	0.28	0.19	0.23
2	4	17	5	12.5	9	2
3	0.58	0.58	1.33	0.75	0.08	0.5
4	1	9	5.5	26.5	1	0.5
5	0.4	0.8	0.2	1.4	9.6	1.2
6	0.36	0.44	0.56	0.28	0.14	1.17

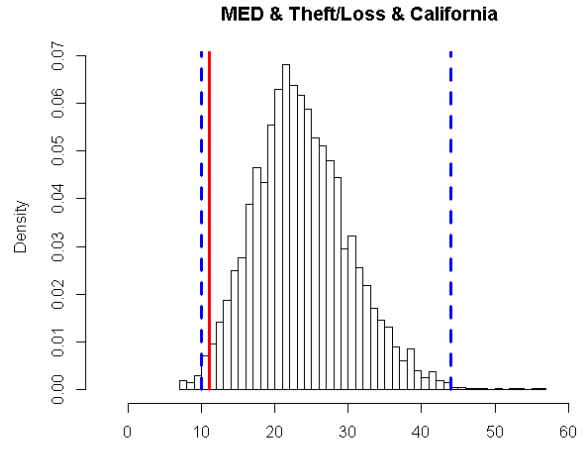
Table 10: Ratios Maximum excitation/basic intensity $(\frac{\Gamma_{i,j}}{\mu_0^{(i)}})_{1\leq i,j\leq 6}$

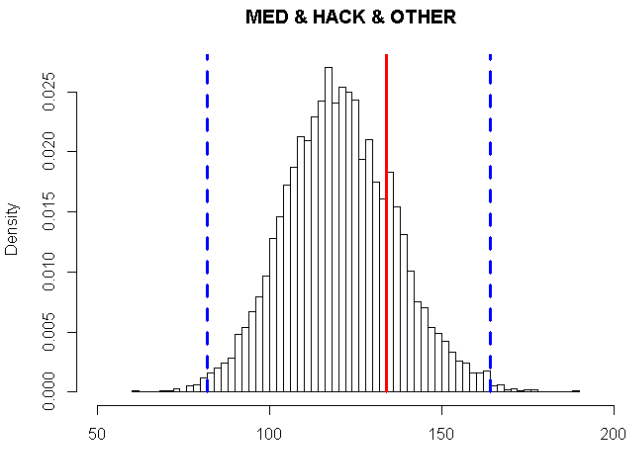
Out-of-sample prediction results for 2017 (kernel 3)

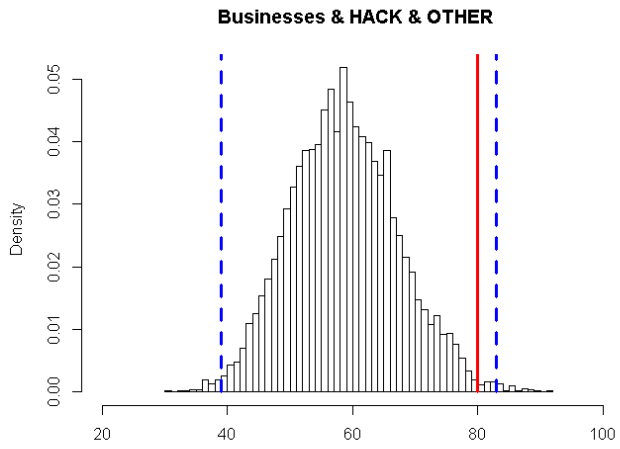












Simulation based on the thinning algorithm for point processes

Predictions with mean and (0.5%, 99.5%) percentiles

Joint prediction of all groups capturing the causal and asymmetric interactions

Parameter uncertainty can be added

Conclusion

- **Take-away message**
  - Heterogeneity of the database: the choice of the groups is determinant for the prediction accuracy.
  - Auto/inter-excitation, calibrated using multivariate Hawkes processes (and kernel with delay)
  - Projection: whole joint distribution of the events' arrivals (and not only the marginal distributions)

- **For further study**
  - Risk Exposure
  - Severity of cyber risk: see Sébastien Farkas' talk

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Paper available at https://hal.archives-ouvertes.fr/hal-02546343

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Thank you for your attention

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