Agent Based Models: Dynamics, Stochastics and Rule based Decisions - A Model Study

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Abstract

One of the first in the actuarial literature published agent based models (ABM) is by Ingram et al. [1]. The paper describes a model of a competitive (insurance) market that shows cyclical behavior. The authors put their focus on the model's theoretic foundation within the theory of plural rationality and on a brief, tabulated, code-like description of the model.
We reformulate the above cited model in a form that makes it accessible for analytical as well as numerical treatment and discussion. We find three, interacting components of the model: the dynamics, the stochastics and the rule based decisions. The agents, insurance companies, play a rule based strategic game, competing with each other. The actions of the agents depend on both, the statistics of the single agent and the statistics of the market as a whole. We analyze the dynamics of the model being responsible for a parameter dependent, periodic behavior and investigate its stochastic and rule-based components. We implemented the model as a Monte Carlo simulation. Therefore we are able to examine the interactions of the model's different components. Finally, we discuss the result of the model as well as possible applications.

Keywords:
Agent based model, rule based decision, insurance market.

1 Introduction

We start with a short introduction of ABMs according to [2]. An ABM consists of a large amount of agents – large in the sense that one agent can not determine the whole system. In this paper an agent represents an insurance company. The agents are defined on a micro level and effect as a whole the properties and the dynamic behavior of the entire system on the macro level. (see Fig. 1)
Every agent is characterized by its own special features:

- Internal degree of freedom (internal parameter): These are individual characteristics of each agent that characterize it. These properties can change over time. (dynamic model).
- Autonomy: The action of each agent depends on its properties (internal parameter) and the state of the "environment“. The "environment" can be given from the outside (external parameters) or results from statistical quantities of the other agents (e.g. the nearest neighbors or the average value of the total collective of agents).
- Mobility: Every agent can operate at the micro level. It can act reactively (on other agents or the environment) or proactively (influencing other agents and the environment).

The interaction between the agents is rule-based. These rules are set a priori and may change over time. The principle of locality applies, i.e. the influence of a single agent is small compared to the whole system. It is examined how the overall system behaves on the macro level and what (statistical) properties it exhibits. The effects of the properties and rules at the micro level (agents) on the overall system are studied and evaluated.
What is often criticized about ABMs is that the properties of the model cannot be clearly assigned to the effects they generate. We therefore revisit the model by Ingram et al. [1] in terms of its structured components and we analyze the effects of the model within this framework. The above cited model by Ingram et al. is one of the first, in the actuarial literature published ABMs. The paper describes a model of a competitive (insurance) market that shows cyclical behavior. The authors put their focus on a brief, tabulated, code-like description of the model. We reformulate the model in a form that makes it accessible for analytical as well as numerical treatment and discussion.

2 The Model revisited

The smallest entity of the model, the **single agent**, represents an insurance company. The number of the agents is n (n = 30 in [1]). Every agent has the following internal degrees of freedom:

$$C_{jt} \quad : \quad Cash \text{ of } company$$
$$K_{jt} \quad : \quad Capital \text{ of } company$$
$$B_{jt} \quad : \quad Belief \in \{Pragmatist; Conservator; Maximiser; Manager\}$$

With the following definitions:

n	:	*Total number of companies*
j	$=1,2,...,n$	*Index of companies*
n_b	:	*Number of bankrupt companies*
T	:	*Number of timestps*
t	$= 0,1,2,...,T$	*Time index*

A company (agent) is bankrupt as soon as $C_{jt} < 0$.

The time dynamics of every single agent is given via the following recursive formulation:

$$\begin{aligned} K_{jt+1} &= K_{jt} + i_{jt} \cdot C_{jt} \\ C_{jt+1} &= C_{jt} + r_{jt} \cdot K_{jt} \end{aligned} \qquad (2.01)$$

With the investment rate $\boldsymbol{i}$ and the return $\boldsymbol{r}$, both depending on the time index t and on the internal parameters of agent j as follows. The system of equation means that the agent invests a ration $\boldsymbol{i}$ of its cash and adds it to its capital. The asset return with rate $\boldsymbol{r}$ is paid to the agent's cash account. The investment rate i_{jt} depends only on the agent's belief B_{jt} (an investment rate between 0% and 30% is assigned to each specification of "belief". see [1], p.408), whereas the return depends on both, the agent's belief B_{jt} and the state of the environment e_t (see below). The return is stochastically sampled from a uniform distribution according to:

$$r_{jt} \propto U\left[0.95\rho_e\left(B_{jt}, e_t\right); 1.05\rho_e\left(B_{jt}, e_t\right)\right] \quad (2.02)$$

where the parameter ρ_e depends on the two dimensional state space of the environment with values as given in [1] p.408. agent can change its belief after three years. On the basis of every single agent the following statistical quantities are calculated within the model:

$\overline{r}_{jt}^{(3)}$: the past three years' mean of the company's returns. (2.03)

$s_{jt} = \sum_{i=0}^{2} sign\left(r_{jt-i}\right)$: the sum of the return's signs of the last three years. (2.04)

Both quantities are important parameters for the decision rules as will be defined in the following. After three years every agent (company) can change its belief. The change obeys very complicated rules that depend only on the statistical quantities of the single agent (2.03 and 2.04) and of the whole system (2.08 and 2.07). For details see [1] pp. 409/410.

The **environment** of the ABM at hand has two components: the first one is the bank. It holds the cash and pays the returns of the companies' investments. It obeys the following equation of motion

$$BC_{t+1} = BC_t - \sum_j r_{jt} \cdot K_{jt} \qquad (2.05)$$

and defines the whole ABM as a closed system in the sense that

$$BC_t + \sum_j C_{jt} = BC_0 + \sum_j C_{j0} = const. \qquad (2.06)$$

Secondly, the environment is defined via an environmental state space e_t:

$$e_t : \{uncertain; bust; boom; \text{mod}\,erate\} \otimes \{a, b\} \quad (2.07)$$

that changes over time. The environmental state space consists of two components: the state of the economy as a whole, i.e. the statistical ensemble of all agents, with the specifications "uncertain", "bust", "boom", "moderate" and secondly, a randomly drawn sub-environment a or b.
On the basis of the whole system the following statistical quantities are calculated within the model:

$\langle \bar{r} \rangle_t^{(n)}$: the mean return of all n companies for year t. (2.08)

$\langle \bar{r} \rangle_t^{(top5)}$: the mean return of the top 5 companies for year t. (2.09)

Both quantities are important parameters for the decision rules defining the development of the economic environment as follows.
The first component of the environmental state space e_t can change every year and depends on the following quantities: the bank's cash, the sum of the companies' capitals and the number of bankrupt companies. The rules are given by:

- $BC_t > \sum_j K_{jt}$ → „boom"
- $BC_t \leq 0$ → „moderate"
- $BC_t \leq 0$ in three out of four successive terms → „uncertain" (2.10)
- $n_b \geq 0.2n$ → „bust"

Now we summarize the three, interacting components of the model: the dynamics, the stochastics and the rule based decisions:

Dynamics:
- The equations of motion for the cash and the capital of the company (eq. 2.01)
- The equation of motion of the bank (eq. 2.05) and the constraint of the closed system (eq. 2.06).

Rule-based decisions:
- The change of the belief of the single agent.
- The change of the first dimension of the two dimensional environmental state space.

Stochastics:
- The return of the companies according to the uniform distribution (eq. 2.02)
- The second part of the environmental state space, choosing randomly between two different sub environments a and b (eq. 2.07).

3 Analysis of the Model

First, we investigate the free motion of the agents without the constraints that puts the environment on them: The system of equations (2.01) can be written as:

$$\begin{pmatrix} K \\ C \end{pmatrix}_{t+1} - \begin{pmatrix} K \\ C \end{pmatrix}_{t} = J \begin{pmatrix} K \\ C \end{pmatrix}_{t} \qquad (2.11)$$

with the matrix $$J = \begin{pmatrix} 0 & i \\ r & 0 \end{pmatrix}. \qquad (2.12)$$

The Eigenvalues of $\boldsymbol{J}$ are given by $\lambda = \pm\sqrt{r\,i}$. As the investment rate $\boldsymbol{i}$ is always not negative ($i \geq 0$), we have, according to the theory of differential equations, exponential behavior if the return is not negative ($r \geq 0$), otherwise oscillations with the frequency $\omega = \sqrt{|r|\,i}$. Fig. 2 shows examples for these two types of solutions for the capital $\boldsymbol{K}$ and the cash $\boldsymbol{C}$ of the company for 100 timesteps. Depicted are the capital (dark color) and the cash (light color) in the exponential case (right) with parameters $i = 1\%; r = 5\%$ and in the oscillating case (left) with $i = 15\%; r = -10\%$ and a thereof resulting period length of $T = \frac{2\pi}{\omega} = 51{,}3$ (compare with Fig. 2). In both cases we have chosen the initial conditions $C_0 = K_0 = 50$ as can be seen in Fig. 2.

This free motion is prohibited in the model [1] because of the following two restrictions:

- If the cash of a company becomes negative, the company is bankrupt and the remaining capital is given back to the bank This influences the oscillating type of the trajectories decisive: only the part is allowed where $C > 0$. In the case that is depicted in Fig. 2 left, both trajectories and the agent would find its end shortly before reaching timestep 10.

- The second restriction is caused by the closed system constraints (2.05) and (2.06). This means that the sum of the cash accounts of the companies and the bank is constant. This effects both types of motion, especially the exponential type, in the sense that there is an upper boundary for the trajectories: due to the lack of cash they can not infinitely grow. In the contrary to the first point this is a systemic effect and not one of the single agent. Nevertheless, every single agent is affected as soon as the boundary is exceeded. In this case agents vanish, new agents with maybe other internal parameters appear and the environment changes according to (2.10).

To investigate these two effects, we simulate the behavior of agents of all four „belief"-types under different return scenarios. We start with equally weighted (comparable to equally distributed number of agents across the four "beliefs") and equally capitalized agents with initial conditions $C_0 = 60$ and $K_0 = 40$ in all four cases. As in the original paper [1] we chose the following investment rates: $i_1 = 5\%, i_2 = 0\%, i_3 = 30\%, i_4 = 15\%$. Where the index 1 corresponds to the belief "Pragmatist", the index 2 to "Conservator", the index 3 to "Maximiser" and the index 4 to "Manager". We use the same nomenclature for the indices of returns in the following. We compare a case where all agents show exponential growth ($r_1 = 10\%, r_2 = 0\%, r_3 = 20\%, r_4 = 15\%$, see Fig. 3 left) with a case where exponential and oscillating behavior are mixed ($r_1 = 10\%, r_2 = 0\%, r_3 = -20\%, r_4 = -15\%$, see Fig. 3 right). In the second case the returns of "Maximisers" and "Managers" are negative and therefore exhibit oscillations. In Fig. 3 the time development of the agents' cash (light colors) and capital (dark colors) is depicted in the upper graphs, the time development of the bank's cash is depicted on the lower graphs. In the upper graphs we use the following color code for the agents' different beliefs: blue for "Pragmatist", orange for "Conservator", grey / black for "Maximiser" and yellow for "Manager". In the first case, exponential grow for all types of agents, left part of Fig. 3, we see that the cash and the capital start at their initial values as given above and show exponential growth at different rates according to their different, above given investment rates and returns. As the bank has to finance the returns, the agents earn on their capitals (see eq. 2.01 and 2.05), the cash account of the bank falls down monotonically as can be seen in the lower left part of Fig.3. At latest around timestep 15 decisions have to be taken in order to keep the whole system stable and ongoing. In the second case, exponential growth for two types of agents vs. oscillations for the other two types of agents, right part of Fig. 3, we see that the cash and the capital start at their initial values as given above and show exponential growth for "Pragmatist" and "Conservator" – just the same as for the first case. For the two types with oscillating behavior the company gets bankrupt as soon as the cash has a negative value. In this point of time the bank sets up a new company with cash equal to the remaining capital of the bankrupt one and with zero capital. Fig. 3 shows that the "Maximiser" (grey / black) bankrupts five times and the "Manager" (yellow) bankrupts four times within the given timespan. The negative returns induce a surplus in the bank's cash as becomes clear from equations (2.01), (2.05) and the lower right part of Fig. 3. Additionally, the bankruptcies can be detected as jumps in the bank's cash curve. The effect of negative returns causes therefore a much slower decay of the bank's cash curve as compared with the first case (left side of Fig. 3).
This are only two examples of what should be inquired in order to understand the ABM under investigation in a reasonable way. Such considerations are essential in order to decide on the realism and applicability of the model.

The influence of stochastics in this ABM is limited and mainly contributes to smoothing the results. This is because relatively narrow distributions (see 2.02 and 2.07) are used for stochastic simulation and because the sum of many stochastic components is included in the final result. Therefore, according to the central limit theorem, distributions are near the normal case.
In the original publication there are no analyses and evaluations of the kind we have carried out and discussed in this section. There is only one single stochastic simulation (one single realisation) of the time development of the number of agents. A cyclical pattern can be seen (see [1] p. 399). In order to better understand the model, other essential quantities such as cash and capital must also be considered.

4 Outlook

The model [1] is an ABM that can reproduce cyclical behavior in the insurance industry or in the economy in general. In order to decide whether and to what extent the model can be used reasonably in reality, it needs to be examined and understood more thoroughly. We propose the following points for this purpose:

- Analysis of frequency patterns: There are different triggers for cyclical behavior of the modeled quantities: deterministically dynamic (see eq. (2.11/12)), bouncing against boundaries of constraints (see 2.05/06 and the second example in section 3), the rule based change of the agents' beliefs. The Fourier analysis is a means to understand the frequency distribution and to relate it to the above mentioned effects.
- Each agent can be assigned to a "Beliefs" at any timepoint. Therefore the ABM can be represented as a model that operates on a network. The four classes of "belief" correspond to the nodes of the network. The links represent the transport of agent number, capital and cash between the nodes. The statistical properties of these transport quantities (flows) need to be studied to better understand the model. It must be kept in mind that the environment has a large influence on this investigation and needs to be considered appropriately.

Based on these investigations it can be determined whether the model has realistic properties. If this is the case, reasonable methods for the parameter estimation of the ABM can be found in this way. In addition to the gain in knowledge and the application for forecasts, we see the potential of this ABM in "gamification". With the help of ABMs, self-reflection with regard to one's own behavior in the insurance market should be stimulated and guided in a goal-oriented manner. The ABM represents an alter ego, which is supposed to stimulate the gain of experience and knowledge regarding their own situation and their own decision-making behavior.

References

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Figures

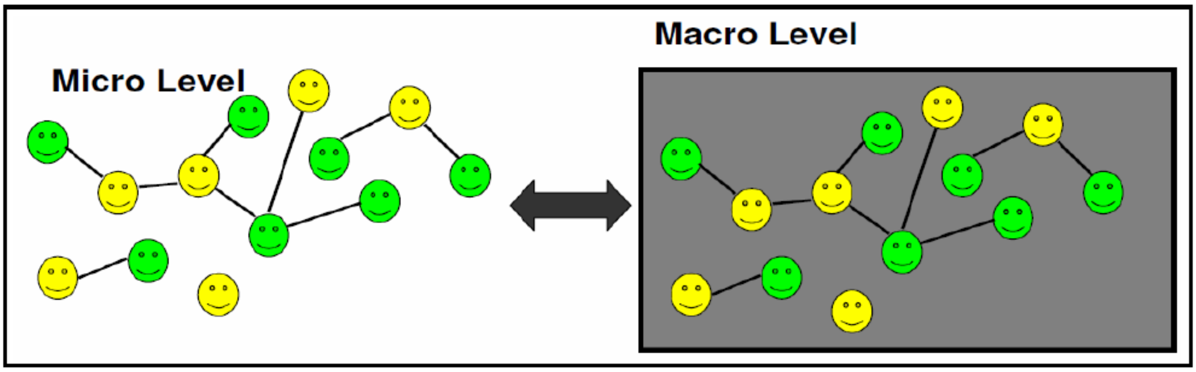

Fig. 1: Systematic visualization of an agent-based model: interaction of macro and micro level

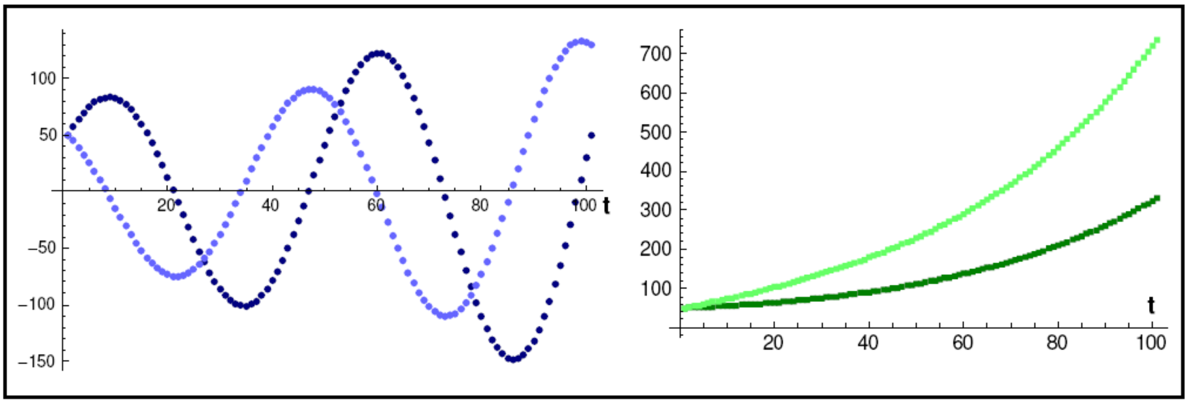

Fig. 2: The development over time of the capital (dark colour) and the cash (light colour) in the exponential case (right, green) with parameters $i = 1\%$; $r = 5\%$ and in the oscillating case (left, blue) with $i = 15\%$; $r = -10\%$.

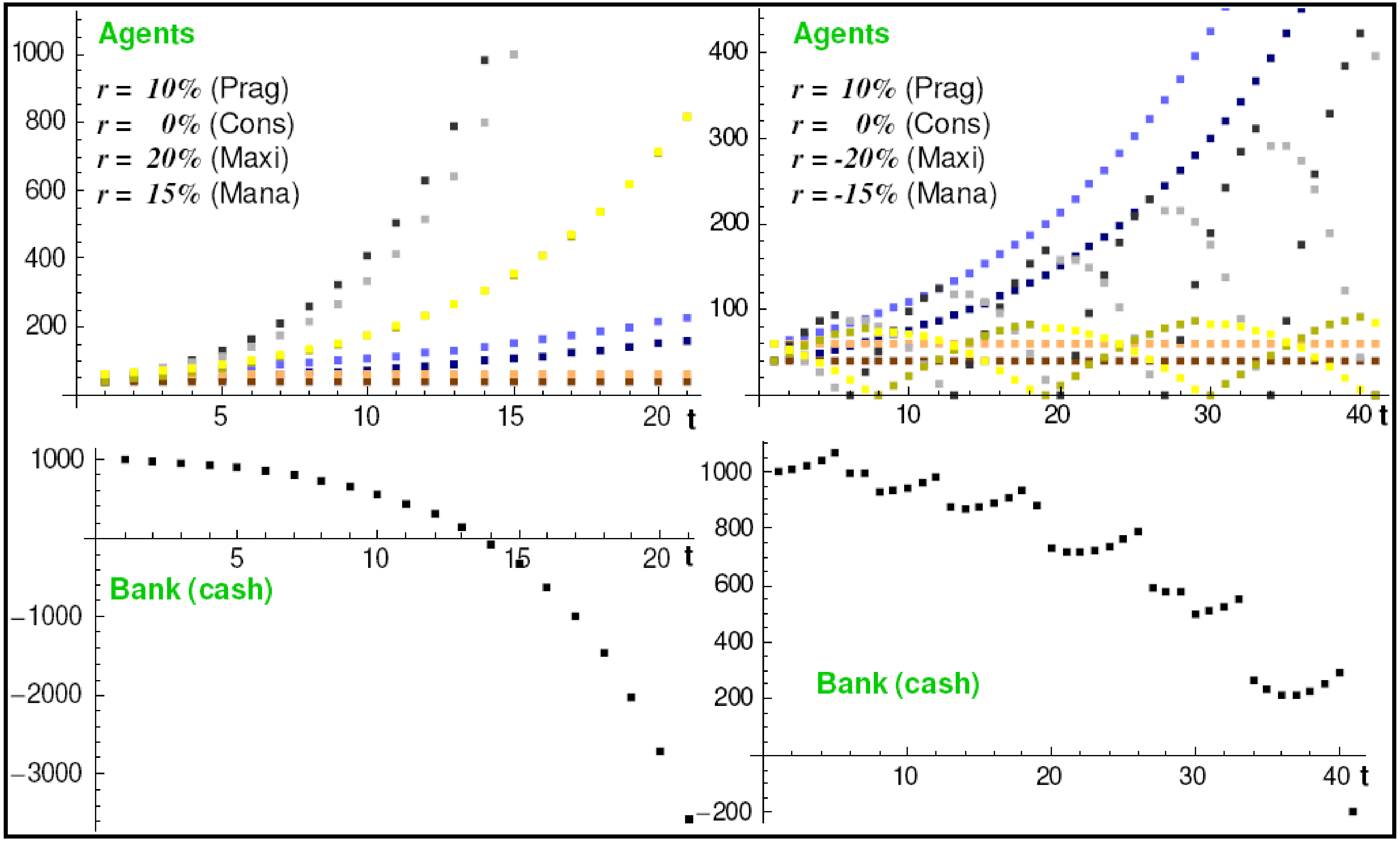

Fig. 3: The time development of cash (light colours) and capital (dark colours) for the 4 types of "belief" (upper) and the cash of the bank (lower) for two different return scenarios.